

Word Problem Notes

- 1) 1000 tickets were sold. Adult tickets cost \$8.50, children's cost \$4.50, and a total of \$7300 was collected. How many tickets of each kind were sold?

System  $8.5A + 4.5C = 7300$   
 $A + C = 1000$

Matrix  $\begin{bmatrix} 8.5 & 4.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} 7300 \\ 1000 \end{bmatrix}$

$\begin{bmatrix} A \\ C \end{bmatrix} = \begin{bmatrix} 8.5 & 4.5 \\ 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7300 \\ 1000 \end{bmatrix}$

$\frac{1}{4} \begin{bmatrix} 1 & -4.5 \\ -1 & 8.5 \end{bmatrix} \cdot \begin{bmatrix} 7300 \\ 1000 \end{bmatrix} = \begin{bmatrix} 700 \\ 300 \end{bmatrix}$

- 2) Mrs. B. invested \$58,855; part at an interest rate of 5%, and part at an interest of 8%. The total interest collected on the investment was \$3,929. How much did she invest at each rate?

System.  $X + y = 58,855$   
 $0.05X + 0.08y = 3,929$

$X = 25,980$   
 $y = 32,875$

Matrix  $\begin{bmatrix} 1 & 1 \\ 0.05 & 0.08 \end{bmatrix} \begin{bmatrix} X \\ y \end{bmatrix} = \begin{bmatrix} 58,855 \\ 3,929 \end{bmatrix}$

$\frac{1}{.03} \begin{bmatrix} 0.08 & -1 \\ -0.05 & 1 \end{bmatrix} \begin{bmatrix} 58,855 \\ 3,929 \end{bmatrix}$

~~$N - 2D = 0$~~   ~~$2N - D = 0$~~   $N = 2D$  or  $D = \frac{N}{2}$

- 3) Samantha has 44 coins, consisting of quarters, dimes, and nickels which total \$7.15. There are twice as many nickels as dimes. How many of each does she have?

$Q + D + N = 44$   
 $25Q + 10D + 05N = 715$   
 $-2D + N = 0$

$\begin{bmatrix} Q \\ D \\ N \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 25 & 10 & 05 \\ 0 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 44 \\ 715 \\ 0 \end{bmatrix}$

$\begin{bmatrix} Q \\ D \\ N \end{bmatrix} = \begin{bmatrix} 23 \\ 7 \\ 14 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 \\ 25 & 10 & 05 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} Q \\ D \\ N \end{bmatrix} = \begin{bmatrix} 44 \\ 7.15 \\ 0 \end{bmatrix}$

4) A boat traveled 168 miles each way downstream and back. The trip downstream took 12 hours. The trip back took 21 hours. Find the speed of the boat in still water and the speed of the current.

$$\frac{168}{12} = 14$$

$$B + C = 14$$

$$\frac{168}{21} = 8$$

$$B - C = 8$$

$$\begin{bmatrix} B \\ C \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 14 \\ 8 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -14 - 8 \\ -14 + 8 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} -22 \\ -6 \end{bmatrix} = \begin{bmatrix} 11 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 14 \\ 8 \end{bmatrix}$$

If the following matrix does not have an inverse, find the value of  $x$ .

$$5) \begin{bmatrix} -5 & 4 \\ 9 & x \end{bmatrix}$$

$$-5x - 49 = 0$$

$$-5x - 36 = 0$$

$$-5x = 36$$

$$x = -36/5$$

Evaluate. Remember, this is the notation for the determinant of a matrix.

$$6) \begin{vmatrix} 5 & 2 \\ 4 & -5 \end{vmatrix}$$

$$-25 - 8 = -33$$

Find the inverse of each matrix.

$$7) \begin{bmatrix} -14 & 21 \\ 4 & -6 \end{bmatrix}$$

$$\frac{-14 \quad 21}{x-6 \quad 4} \quad \frac{21 \quad 4}{4 \quad -6} \quad 84 - 84 = 0$$

No inverse possible

$$8) \begin{bmatrix} -8 & 3 \\ -1 & -1 \end{bmatrix} \quad \det A = 8 - (-3) = 11$$

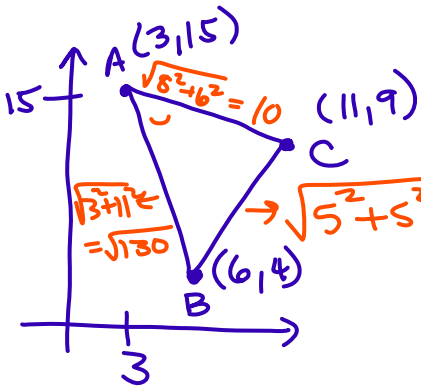
$$A^{-1} = \frac{1}{11} \begin{bmatrix} -1 & -3 \\ 1 & -8 \end{bmatrix}$$

$$A^{-1} \cdot A = I$$

$$\frac{1}{11} \begin{bmatrix} -1 & -3 \\ 1 & -8 \end{bmatrix} \begin{bmatrix} -8 & 3 \\ -1 & -1 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 11 & 0 \\ 0 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

Find the area of a triangle with the given vertices.

9)  $A(3, 15)$ ,  $B(6, 4)$ , and  $C(11, 9)$   $A = \frac{1}{2} ab \sin C$



$A = 37.875^\circ$   
 $\text{area} = \frac{1}{2} \sqrt{130} \cdot 10 \cdot \sin 37.875^\circ$   
 $\text{area} = 35$

$$\frac{1}{2} \left| \det \begin{vmatrix} 3 & 15 & 1 \\ 6 & 4 & 1 \\ 11 & 9 & 1 \end{vmatrix} \right|$$

$$= 35$$

$(\sqrt{50})^2 = (\sqrt{130})^2 + (10)^2 - 2\sqrt{130} \cdot 10 \cos A$

10)  $(-1, 7)$ ,  $(2, 6)$ , and  $(4, -3)$

$$\frac{1}{2} \left| \det \begin{vmatrix} -1 & 7 & 1 \\ 2 & 6 & 1 \\ 4 & -3 & 1 \end{vmatrix} \right|$$

$= \frac{1}{2} \cdot 25$

$= 12.5$

11)  $(-2, 5)$ ,  $(4, 2)$ , and  $(0, 7)$

$$\frac{1}{2} \left| \det \begin{vmatrix} -2 & 5 & 1 \\ 4 & 2 & 1 \\ 0 & 7 & 1 \end{vmatrix} \right|$$

$= 9$

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700 and 300

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\$25,980 at 5% and \$32,875 at 8%

- 3) Samantha has 44 coins, consisting of quarters, dimes, and nickels which total \$7.15. There are twice as many nickels as dimes. How many of each does she have?

23Q and 7D and 14N

- 4) A boat traveled 168 miles each way downstream and back. The trip downstream took 12 hours. The trip back took 21 hours. Find the speed of the boat in still water and the speed of the current.

boat: 11 mph, current: 3 mph

If the following matrix does not have an inverse, find the value of  $x$ .

$$5) \begin{bmatrix} -5 & 4 \\ 9 & x \end{bmatrix}$$

-7.2

Evaluate. Remember, this is the notation for the determinant of a matrix.

$$6) \begin{vmatrix} 5 & 2 \\ 4 & -5 \end{vmatrix}$$

-33

Find the inverse of each matrix.

$$7) \begin{bmatrix} -14 & 21 \\ 4 & -6 \end{bmatrix}$$

No inverse exists

$$8) \begin{bmatrix} -8 & 3 \\ -1 & -1 \end{bmatrix}$$

$$\frac{1}{11} \cdot \begin{bmatrix} -1 & -3 \\ 1 & -8 \end{bmatrix}$$

**Find the area of a triangle with the given vertices.**

9)  $(3, 15)$ ,  $(6, 4)$ , and  $(11, 9)$

35 square units

10)  $(-1, 7)$ ,  $(2, 6)$ , and  $(4, -3)$

12.5 square units

11)  $(-2, 5)$ ,  $(4, 2)$ , and  $(0, 7)$

9 square units